

EXERCISE – IV**ADVANCED SUBJECTIVE QUESTIONS**

$$1. \int \frac{\tan 2\theta}{\sqrt{\cos^6 \theta + \sin^6 \theta}} d\theta$$

$$2. \int \frac{5x^4 + 4x^5}{(x^5 + x + 1)^2} dx$$

$$3. \int \frac{\cos^2 x}{1 + \tan x} dx$$

$$4. \text{Integrate } \int \frac{dx}{x\sqrt{x^2 + 2x - 1}} \text{ by the substitution}$$

$$z = x + \sqrt{x^2 + 2x - 1}$$

$$5. \int \left[\left(\frac{x}{e} \right)^x + \left(\frac{e}{x} \right)^x \right] \ln x \, dx$$

$$6. \int \frac{a^2 \sin^2 x + b^2 \cos^2 x}{a^4 \sin^2 x + b^4 \cos^2 x} dx$$

$$7. \int \frac{dx}{\left(x + \sqrt{x^2 - 1} \right)^2}$$

$$8. \int \sqrt{\frac{\sin(x-a)}{\sin(x+a)}} dx$$

$$9. \int \frac{\cot x \, dx}{(1 - \sin x)(\sec x + 1)}$$

$$10. \int \sin^{-1} \sqrt{\frac{x}{a+x}} dx$$

$$11. \int \left[\frac{\sqrt{x^2 + 1} [\ln(x^2 + 1) - 2 \ln x]}{x^4} \right] dx$$

$$12. \int \frac{x+1}{x(1+xe^x)^2} dx$$

13. Let $f(x)$ is a quadratic function such that $f(0) = 1$ and $\int \frac{f(x) dx}{x^2(x+1)^3}$ is a rational function, find the value of $f'(0)$.

14. Integrate $\frac{1}{2} f'(x)$ w.r.t. x^4 ,
where $f(x) = \tan^{-1} x + \ln \sqrt{1+x} - \ln \sqrt{1-x}$

$$15. \int \frac{(\sqrt{x} + 1) dx}{\sqrt{x} (\sqrt[3]{x} + 1)}$$

$$16. \int \frac{dx}{\sin \frac{x}{2} \sqrt{\cos^3 \frac{x}{2}}}$$

$$17. \int \frac{x^2 + x}{(e^x + x + 1)^2} dx$$

$$18. \int \sqrt{\frac{\operatorname{cosec} x - \cot x}{\operatorname{cosec} x + \cot x}} \cdot \frac{\sec x}{\sqrt{1 + 2 \sec x}} dx$$

$$19. \int \frac{\cos x - \sin x}{7 - 9 \sin 2x} dx$$

$$20. \int \frac{dx}{\sec x + \operatorname{cosec} x}$$

$$21. \int \frac{dx}{\sin x + \sec x}$$

$$22. \int \tan x \cdot \tan 2x \cdot \tan 3x \, dx$$

$$23. \int \frac{3 + 4 \sin x + 2 \cos x}{3 + 2 \sin x + \cos x} dx$$

$$24. \int \frac{e^{\cos x} (x \sin^3 x + \cos x)}{\sin^2 x} dx$$

$$25. \int \frac{(ax^2 - b)dx}{x\sqrt{c^2x^2 - (ax^2 + b)^2}}$$

$$26. \int \frac{e^x (2 - x^2)}{(1 - x)\sqrt{1 - x^2}} dx$$

$$27. \int \frac{x \ln x}{(x^2 - 1)^{3/2}} dx$$

$$28. \int \sqrt{\frac{(1 - \sin x)(2 - \sin x)}{(1 + \sin x)(2 + \sin x)}} dx$$

$$29. \int \frac{4x^5 - 7x^4 + 8x^3 - 2x^2 + 4x - 7}{x^2(x^2 + 1)^2} dx$$

$$30. \int \frac{\sqrt{2 - x - x^2}}{x^2} dx$$

$$31. \int \frac{dx}{(x - \alpha)\sqrt{(x - \alpha)(x - \beta)}}$$

$$32. \int \frac{dx}{\cos^3 x - \sin^3 x}$$